USING PHYSLETS TO TEACH QUANTUM MECHANICS

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Abstract

We have produced interactive Physlet®-based curricular materials to support introductory, intermediate, and advanced courses in quantum mechanics. These exercises, demonstrations, and tools address both quantitative and conceptual difficulties encountered by many students. In addition, these quantum mechanics exercises are appropriate for use with the Just-in-Time Teaching technique to actively engage students outside of the classroom. Examples of the curricular materials, the results of our preliminary assessment of their effectiveness, and future directions of this project will be discussed.

Introduction

Educators have often pinned their hopes of better instruction on emerging technologies such as television, computers and the World Wide Web. Yet teaching with technology, without a sound pedagogy, can yield no significant educational gain [1]. In addition, research on problem solving [2-4] indicates that students often approach typical end-of-chapter problems with little regard to the conceptual underpinnings of the problem. They direct most of their efforts at finding a formula that contains the variables given in the problem statement. We use Physlets combined with Just-in-Time Teaching (JiTT) to fully exploit current technology to create alternative problems that we believe can help students to better develop their problem-solving ability and deepen their conceptual understanding.

Pedagogy

Physlets

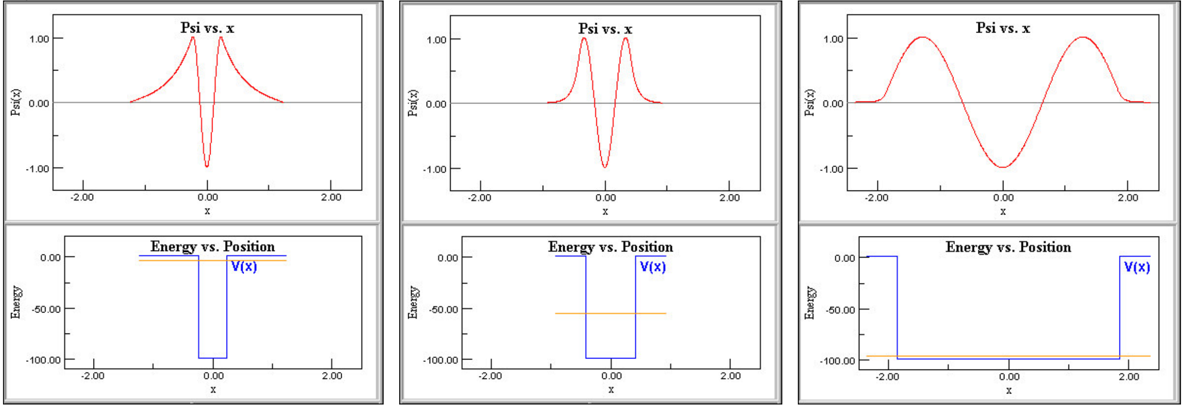

Figure 1: A Physlet-based exercise: students are asked to change the width of the potential energy function and describe what happens to the wave function and the energy levels.

Physlets®—“Physics applets”—are small, flexible Java applets that can be used in a wide variety of WWW applications [5]. Many other physics-related Java applets are being produced around the world—some of them very useful for education. The collection of applets we call “Physlets” has attributes that make it valuable for the educational enterprise:

- **Physlets are simple.** The graphics are simple; each Physlet is designed to deal with only one facet of a physical phenomenon. This keeps Physlets relatively small, which eases download problems over slow networks.

- **Physlets are visual and interactive.** By having the students decide what measurements to make and what variables to change, and by providing them with real-time feedback, students are put in control of the exercise. See for example the finite square well exercise depicted in Figure 1. A visual and interactive exercise enriches student understanding far more than if the physics is just explained on a page of text.
- **Physlets are flexible.** All Physlets can be set up and controlled with JavaScript, meaning the Physlet called *Animator* can be used for almost any subject in mechanics with small changes in the JavaScript—and not the Java—associated with each exercise [6-8].
- **Physlet-based pedagogy is agnostic.** Physlets can be used as an element of almost any curriculum with almost any teaching style. Although we believe that interactive engagement[1] methods [9-11] such as Just-in-Time Teaching [12], Peer Instruction [13], or Tutorials [14] can improve pedagogy, Physlets can also be used as traditional lecture demonstrations and can be given as end-of-chapter homework.
- **Physlets are Web based.** They can run on any platform and are easily embedded in html documents.
- **Physlets are free for noncommercial use.** Physlet archives, that is, compressed archives containing compiled Java programs, can be downloaded from the Davidson College WebPhysics server.

Just-in-Time Teaching

Although the rich content and interactivity provided by Physlets can be pedagogically useful, it can lack the human dimension that is important to effective teaching. To be truly effective, the communication capabilities of the computer must be used to create a feedback loop between instructor and student. A new and particularly promising approach known as Just-in-Time Teaching, JiTT, has been pioneered at Indiana University and the United States Air Force Academy and further developed at Davidson College. JiTT is briefly described below, but the technique is described in detail in *Just-in-Time Teaching: Blending Active Learning with Web Technology* [12].

The JiTT pedagogy exploits an interaction between Web-based study and an active-learner classroom. Students respond electronically to carefully constructed web-based assignments before class. The instructor reads the student submissions “just-in-time” to adjust the lesson content and activities to suit the students’ needs. Thus, the heart of JiTT is the feedback loop—formed by the students’ outside-of-class preparation and the teacher’s outside-of-class preparation in response to student submissions—which fundamentally affects what happens during class time.

Physlet-Based JiTT Exercises

Although JiTT can be implemented fully using technically simple web-based assignments, incorporating Physlet-based exercises heightens the extent to which student understanding can be probed and encouraged. Responding to questions that involve watching, or analyzing, an animation often requires different skills and a different level of understanding than responding to static questions. JiTT is ideally suited to help students improve their analysis skills and deepen their conceptual understanding.

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[1] Hake [9] defines interactive engagement as, “methods as those designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors.”

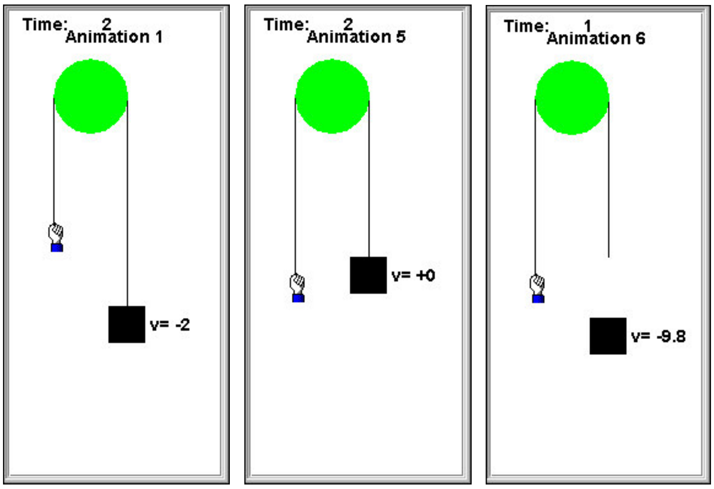

Figure 2: A Physlet-based Just-in-Time Teaching exercise about acceleration and tension. Three of six scenarios are shown.

Figure 2 represents a Physlet-based JiTT exercise for introductory physics students. There are six animations that students are asked to rank [15] according to the acceleration of the black mass and the tension of the rope. Students must watch the animation to determine the acceleration of the mass and use this information to calculate the tension in the rope. Our experience with this exercise has shown that students do not have any trouble with the acceleration question, but they do have significant trouble with the tension question. As a consequence we end up spending much more class time discussing the tension question and very little class time discussing the acceleration question.

Assessment of Physlet-Based Material in Introductory Mechanics

Over the past four years we have given the Force Concept Inventory [16], FCI, as a pre-test and post-test to 10 of our first-semester introductory (algebra-based and calculus-based) physics classes at Davidson College. The results were tallied and the course average normalized gain was calculated [9, 13]. Results from a study of over 6,000 students [9] show that traditionally taught students have a gain of 0.25 on the FCI. However, students taught in a more interactive way have a significantly greater gain between 0.38 and 0.68.

Figure 3 shows the correlation between the gain versus the number of interactive exercises (using Physlets and JiTT) assigned in the course. In every first-semester introductory physics class, students must complete 10 interactive JiTT pre-lab exercises. Hence the base number of interactive exercises is 10. All courses except the ones with the lowest two gains ($g = 0.29$ and 0.35, the more traditional courses) used either Just-in-Time Teaching (7 courses with $g = 0.53 \pm 0.09$) or Peer Instruction (one course with $g = 0.48$). Just-in-Time Teaching gives results that are comparable to Peer Instruction and both methods give relatively high gains[2].

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[2] Hake [9] would call our gains in the “medium-g” category as he considers gains above 0.7 as “high-g”. However, in Ref. [9] there are no courses with gains above 0.7.

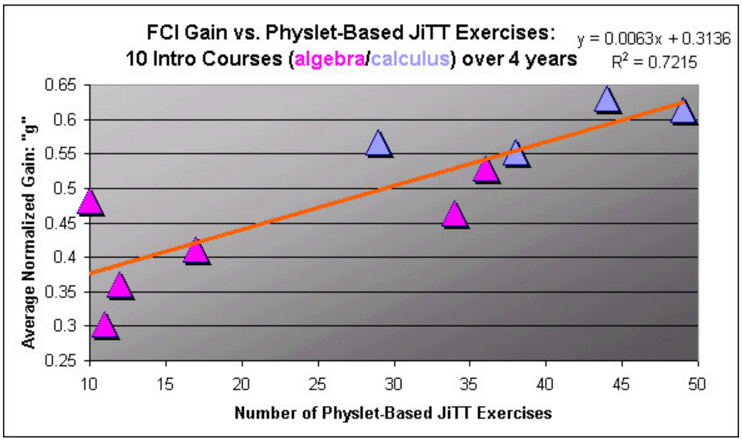


Figure 3: Data from four years and 317 students at Davidson College.

Quantum Mechanics

Curricular Materials for Quantum Mechanics

We have completed and class tested about 50 Physlet-based quantum mechanics exercises in support of the Just-in-Time Teaching approach (a 1-semester junior/senior-level quantum mechanics course). In order to support various teaching pedagogies, we are developing curricular material in three formats:

- Illustrations are interactive essays that animate what would normally be a static figure or a formula in a textbook. They illustrate a physical concept through the integration of an animation and a narrative.
- Explorations are directed interactive tutorials exploring a particular concept. Students are asked several questions and shown an animation that guides them in determining what to do and what to measure.
- Problems are interactive versions of traditional questions that appear at the end of a chapter in most textbooks. They require either a numeric or a conceptual answer.

This material will become the *Interactive Quantum Mechanics Companion* (2003 Prentice Hall), but the original materials created for JiTT are—and will continue to be—freely available for teachers [17]. Below we describe a few exercises.

Time-Independent Exercises

There are several important features of wave functions that students have a hard time understanding. Most of this confusion lies in the fact that the standard examples presented to students are the infinite square well and the quantum harmonic oscillator potential. These potential energy functions are symmetric about the middle of the well and therefore do not provide a rich enough testing ground for quantum mechanical concepts regarding the wave function.

Consider the example shown in Figure 4 where a wave function is shown to students and they are asked:

a) What energy state is depicted?
b) The wave function changes "wavelength" and amplitude, what do these changes mean?
c) What does the potential energy function look like?

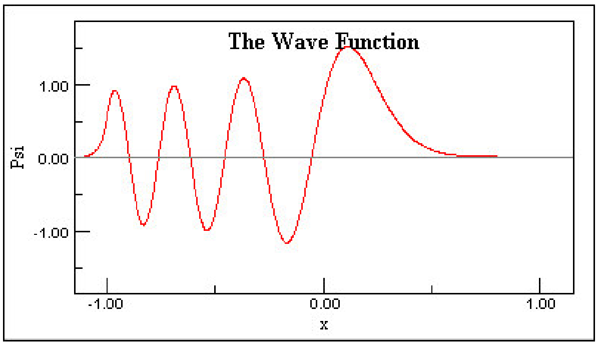
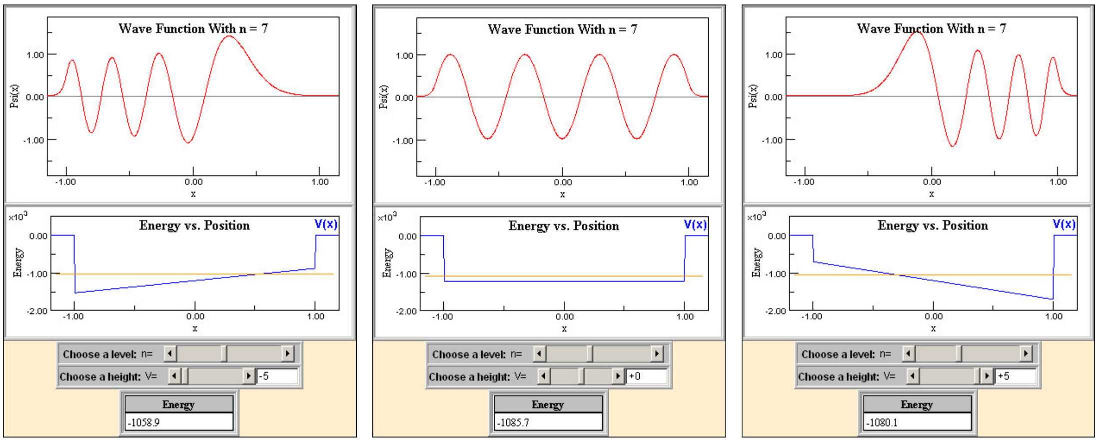


Figure 4: A wave function of an "unknown" potential.

In this exercise notice that the wave function must correspond to the seventh bound state of a ramped potential energy function. This exercise is based on "one of the best quantum mechanics problems ever posed" [18]. Unlike the classic problem [19] in which the potential energy function is shown along with an incorrect wave function, we give students a set of open-ended questions.

We give this Physlet-based JiTT exercise pre-instruction and we follow it with the in-class Illustration depicted in Figure 5.



Choose a level: n=
Choose a height: V= -5
Energy -1058.9

Choose a level: n=
Choose a height: V= +0
Energy -1085.7

Choose a level: n=
Choose a height: V= +5
Energy -1080.1

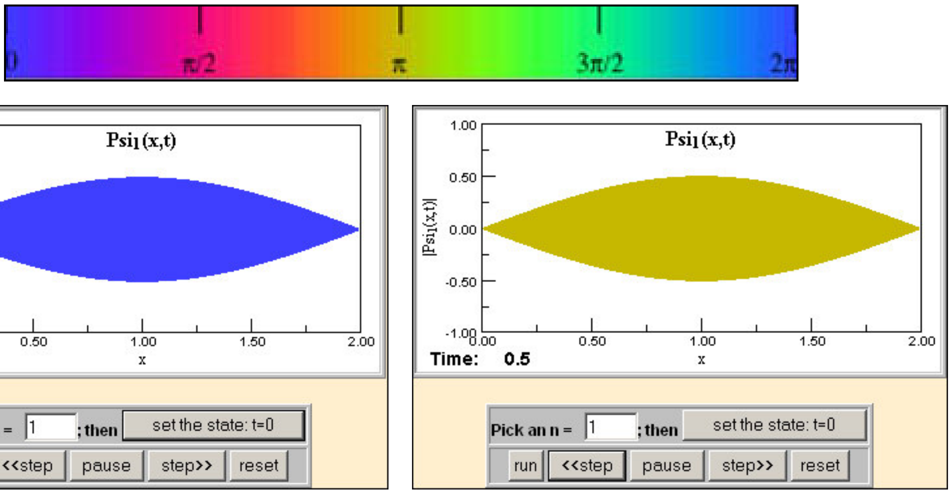
Figure 5: The ramped finite potential energy well Illustration.

In the Illustration both the potential energy function and the wave function are shown. Sliders on the html page allow the student or the instructor to change the "ramping" of the potential energy function and then immediately see the consequences on the wave function.

One important misconception elicited by this Illustration is that the energy of the particle is changing across the well. Since these are energy eigenstates, the energy is, of course, the same across the well. This misconception probably comes from the infinite square well problem. Since the potential is constant (and zero) for the infinite square well problem, changing the curvature of the wave function means that the total energy changes (you go from one energy eigenstate to another). However with the ramped potential energy function, this is not the case. The potential energy function changes and therefore the curvature of the wave function must change in order to keep the total energy the same across the well.

Time-Dependent Exercises

One of the most intriguing problems from an animation or simulation point of view is that of quantum mechanical time development. How one decides to portray complex wave functions in time evolution is an important consideration and various authors have chosen slightly different conventions [20-23]. We show either the separate real and imaginary components or the amplitude of the wave function and the phase of the wave function as color much like other authors [21, 22].



Time: 0

Pick an n = 1 ; then

Time: 0.5

Pick an n = 1 ; then

Figure 6: The time-dependent infinite square well ground state wave function at t = 0 and t = 0.5, where t = 1 is the ground state revival time. Also shown is the correspondence between the color of the wave function and the phase of the wave function.

Figure 6 shows our first time-evolution exercise which represents the phase of the wave function as color. Infinite square well states are chosen because of students' familiarity with these states. Students are asked to play the animation and describe the features of the wave function. At t = 0, depicted in Figure 6, the wave function vanishes at x = 0 and x = 2 as it must, but it is also a maximum at the center of the well with a value of 1. To match up this fact with the above representation, the height of the wave function must be the amplitude.[3]

The color of the wave function corresponds to the phase of the wave function. Since any complex number can be represented as $Ae^{i\theta}$ where A is the amplitude and θ is the phase angle from the real axis. Since quantum mechanical time evolution has a minus sign in the exponential, the time-dependent wave function must rotate clockwise in the complex plane. In the animation t = 1 corresponds to the revival time for the wave function, therefore a real wave function at t = 0 evolves to become negative and imaginary at t = 0.25, then negative and real at t = 0.5, then positive and imaginary at t = 0.75, before becoming real and positive again at t = 1.

Since students do not often "see" the time development of the wave function, they have no reason to believe that there may be a difference between the time evolution of stationary states and a superposition of states. In fact, since the time-evolution operator is a unitary operator it usually drops out of calculations for the probability density. A superposition of states is different however. While it is still true that $< \Psi | \Psi > = 1$ independent of time, the probability density does vary with time as shown in Figure 7.

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[3] There are two standard representations for the complex wave function. Figure 6 shows one. In the other representation the amplitude is always shown as positive. We prefer the representation shown in this paper since it seems to better represent scattering problems.

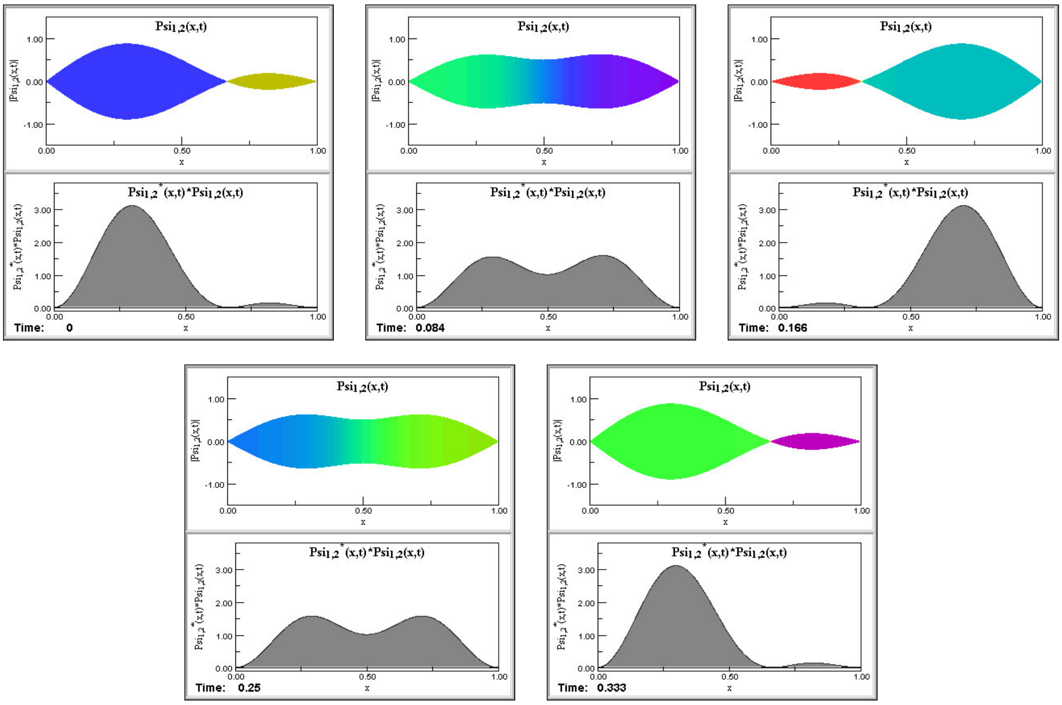


Figure 7: The time evolution of an equal mix superposition of ground state and first excited state wave functions for the infinite square well.

The frequency of oscillation for the probability density is proportional to the difference in energy between the two states in the superposition. However the frequency of oscillation for the wave function is the time it takes for both phases to get back to zero, which is the ground state revival time [24] of 1 for this animation.

Barrier Problems

We begin the discussion of quantum mechanical barrier problems by stressing the similarities and differences between a classical electromagnetic wave incident on a change of index of refraction and a quantum mechanical plane wave incident on a change in potential energy as shown in Figure 8. The animation shows one important feature of the wave in the left medium: when you add up both the left-moving and the right-moving wave, the resulting wave resembles a standing wave.

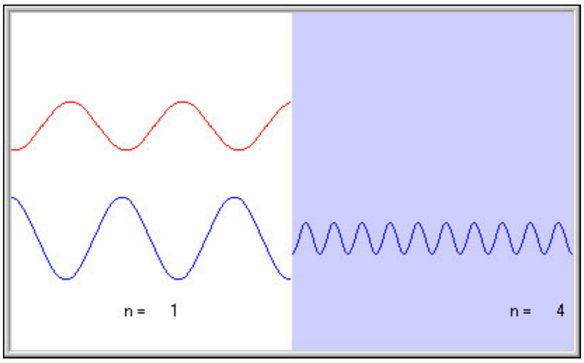
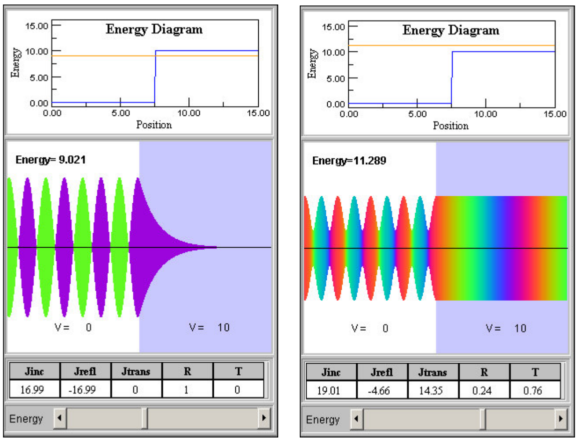


Figure 8: A classical electromagnetic wave traveling to the right through two media (1 and 2) of different indices of refraction. Shown in red is the left-moving part of the EM wave.

Students then see a quantum mechanical plane wave traveling to the right through a change in potential energy as depicted in Figure 9. The energy diagram is shown along with the wave function. The current densities and reflection and transmission coefficients are shown in a table. Multiple representations are important as students often think that the plane wave loses energy as it passes into the barrier.



Jinc	Jrefl	Jtrans	R	T
16.99	-16.99	0	1	0

Jinc	Jrefl	Jtrans	R	T
19.01	-4.66	14.35	0.24	0.76

Figure 9: A quantum mechanical plane wave traveling to the right through a change in potential energy. Shown in the upper graph is the energy diagram.

Note that the amplitude of the transmitted plane wave for E > 10 is larger than the incident plane wave indicating a smaller kinetic energy and therefore a larger potential energy there.

Assessment of Physlet-Based Material in Quantum Mechanics

During the fall 2001 quantum mechanics course, 4 senior physics majors (who all happened to be women) were assigned 16 Physlet-based JiTT exercises [17] during 28 class meetings. They were given the Quantum Mechanics Visualization Instrument [25], QMVI, during the first week of the semester (the pre-test) and then at the end of the semester (the post-test). The QMVI probes conceptual and visual understanding of quantum mechanics through 25 multiple-choice questions regarding classical and quantum probability, wave function shape and potential energy functions, infinite square well, 1d scattering, spin, momentum space, and time development of Gaussian wave packets.

Figure 10 shows the results (out of 100) presented in Ref. [25] for modern physics (28.5), undergraduate quantum mechanics (51.1) and graduate quantum mechanics (55.5) next to the Davidson College undergraduate quantum mechanics results (66.25). The gain for this class was 0.62. Students completed our quantum mechanics course with a conceptual understanding of quantum mechanics at or above the level of graduate students after they had taken a graduate course in quantum mechanics. Because the pre-test scores are so low (11), we see this gain as evidence that our students' conceptual understanding is due to the interactive nature of the quantum mechanics course.

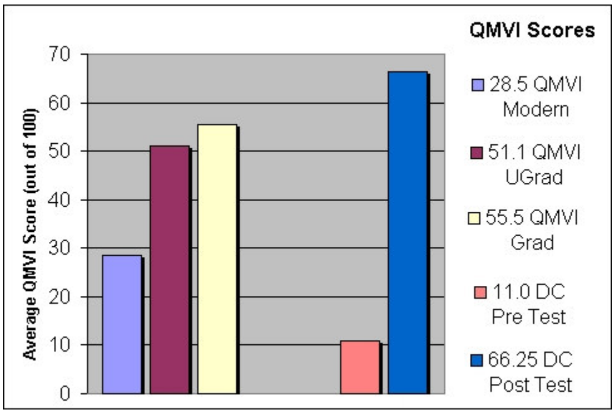


Figure 10: Performance of the fall 2001 Davidson quantum mechanics class on the QMVI.

Conclusion

We have created over 50 Physlet-based exercises for the teaching and learning of quantum mechanics. These materials provide a new, exciting, and effective way to deliver interactive curricular material to students in advanced physics courses. The effectiveness of these materials is supported by our preliminary assessments of the use of interactive curricular material in both introductory physics and quantum mechanics courses. We are currently writing and teaching with new material that will eventually appear as the *Interactive Quantum Mechanics Companion* (2003 Prentice Hall).

Acknowledgements

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